Journal of Applied Mechanics and Technical Physics, Vol. 37, No. 2, 1996

TRANSITION TO SUPERSONIC VELOCITY IN EVAPORATION AND INJECTION FROM A CYLINDRICAL SURFACE INTO VACUUM

M. Yu. Plotnikov and A. K. Rebrov

UDC 533

In the last few decades two aspects of the problems of evaporation have been the center of attention of specialists in rarefied gas dynamics:

(1) Study of the rarefied gas flow structure ($Kn > 0.01$), where the Knudsen number Kn characterizes the conditions of rarefaction either by the size of the limited evaporating surface or by the distance between the evaporating and condensing surfaces;

(2) Determination of boundary conditions for the equations of a continuous medium with intense evaporation, i.e., determination of discontinuities of the Knudsen layer parameters.

Even in the simplest case of homogeneous gas flow, these problems have a complicated physical and mathematical context, which gave rise to a number of various theoretical approaches to the choice of kinetic boundary conditions and also to the methods for solution of the Boltzmann equation. Without making a complete list, we shall mention some studies which differ from one another either by selection of boundary conditions or by methods for solving the Boltzmann equation. The Boltzmann equation was solved using the moment method [1–5], numerical methods including the Monte Carlo method for determining the collision integral [6], iterative algorithms of integration [7], analytical methods based on Hilbert's expansion of the distribution function [8], analytical methods for solving the Bhatnagar–Gross–Krook (BGK) equation [9–11], and numerical solutions of the BGK equation [12–19]. Use of the Boltzmann–Krook–Welander (BKW) equation [20] gives the same results as the BGK equation. The Mott–Smith method was employed in some works [21, 22].

Using a variational approach, Cipolla et al. [23] determined temperature and pressure discontinuities near the surfaces for the linearized Boltzmann equation. To obtain boundary conditions for the equations of continuous medium, Labuntsov and Kryukov [24] performed an analysis of the Knudsen layer near the evaporating surface by solution of the Boltzmann equation using the moment method assuming a bimodal distribution function.

One of the most comprehensive studies of evaporation (condensation) processes was carried out by Sugimoto and Sone [25], where stationary evaporation was studied numerically using the solution of the BKW equation; a new difference scheme has been developed for the detailed investigation of the Knudsen layer, the behavior of gas parameters and the asymptotic limiting cases were studied in a wide range of Knudsen numbers ($Kn \geqslant 0.01$). This paper sums up the long-standing investigations of Sone and coworkers.

The comparative analysis of all the results obtained, the evaluation of the methods, and also the description of the evolution parameters in the vicinity of the evaporating surface using results of particular works is a difficult problem, which is, in addition, not important. The study of the Knudsen layer using methods of kinetic gas theory may be considered completed. But the method of statistical modeling gives some new opportunities. Sibold and Urbassek [26], proceeding from the experiments of Faubel, Schlemmer, and Toennies [27] and using the Bird method, analyzed the behavior of parameters in the vicinity of a cylinder at $Kn = \infty$, 2, and 0.002 and drew some important conlusions concerning not only the description of the

Institute of Thermophysics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090.
Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 37, No. 2, pp. 120–130, March–April, 1996. Original article submitted March 28, 1995.

 0021-8944/96/3702-0250 $15.00

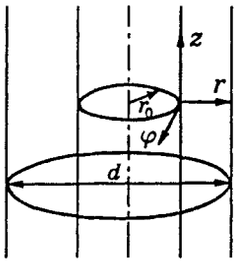

Fig. 1.

evaporation process, but also the history of previous investigations. It seems likely that the method of direct statistical modeling was first applied to evaporation and condensation by Murakami and Oshima [28], who analyzed recondensation within a plane layer and obtained some results which were in agreement with the results of other authors. The problem of intense recondensation of gas between circular coaxial cylinders was solved using direct statistical modeling for a homogeneous gas [29] and for recondensation in the presence of a noncondensing gas [30].

Although the investigation of evaporation from bodies of limited size appears to be completed, little attention has been given in the literature to the question of transition through the sonic speed in the neighborhood of an evaporating surface during expansion into vacuum or into a low-density medium. The fundamental importance of this question lies in the fact that the transition proceeds within an expanding flow but the role of "maxwellization," viscosity, and thermoconductivity in the formation of a sonic surface under a unidirectional geometrical influence has not been completely elucidated. Concerning the sonic surface location, the conclusion can be drawn from the available works, except for [26], that a sonic surface is located either within the Knudsen layer or on its boundary.

In this paper we assume that a surface with a local Knudsen number $\mathrm{Kn}_l = (l/p)(dp/dx) = 0.01$ is the boundary of the Knudsen layer (here p is a thermodynamic parameter that is most sensitive to the boundary, of influence of rarefaction). Beyond this boundary the Navier–Stokes approximation can be used for cases of practical imporantance. From this standpoint, use of the thickness of the Knudsen layer as a scale quantity can most likely be justified by demands of the mathematical model, as in [5], where it was shown that, for plane flow, the Knudsen layer, which is defined formally, can stretch from the evaporating surface to infinity.

In the present paper, the formation of a sonic surface is studied using a more general formulation of the problem than only evaporation, i.e., the case of gas injection from the surface of a cylindrical source into vacuum is analyzed. This formulation supplements the set of possible gas flow conditions with a free selection of the injected gas parameters.

1. Formulation of the Problem. A particle flow is evaporated or injected from the surface of a cylindrical source of radius r_0 and of infinite length into the surrounding medium, which either has a zero pressure at infinity or is enclosed in a concentric cover of diameter d with condensation coefficient $\alpha = 1$ (Fig. 1).

The velocity distribution function for evaporated or injected molecules is specified in the ordinary way. In this formulation (with complete gas condensation either at infinity or at a finite distance) the evaporation coefficient is of no concern for the problem while the condensation coefficient at the evaporating surface can be varied. The conditions of complete condensation at the outlying boundary were specified to eliminate any influence of the residual gase upon the processes that take place near the evaporation (injection) surface. This question requires a special analysis. At a constant flow rate of vapor or injected gas, the conditions at infinity or at the condensing surface must not affect the formation of flow near the source. These considerations were taken into account in the selection of downstream boundary conditions.

The aim of the present work is to find the radial distribution of the vapor (gas) parameters over the expansion zone and to determine the location of the sonic surface.

2. Algorithm of Direct Statistical Modeling. To describe the motion of particles, we introduce a

six-dimensional space $X \times V$, where the particle state is defined by its three spatial coordinates and also by its three velocity components. We define the distribution function $f(x, v)$ in the following way: $f(x, v)dxdv$ is the expected number of particles within volume element dx of the physical space near point x and the velocities of the particles fall within the range dv near point v in the velocity space. Considering the geometry of the problem (Fig. 1), it is convenient to use cylindrical coordinates in the following manner: the z-axis coincides with the cylinder axis; r is the distance to the cylinder axis, and φ is the azimuthal angle. The notation v_r, v_z, v_φ, T_r, T_z, and T_φ denote the gas velocity and temperature components. Under these conditions we assume that the stationary gas flow is described by the Boltzmann equation [31]

$$v_r \frac{\partial f(x,v)}{\partial r} + \frac{v_\varphi}{r}\frac{\partial f(x,v)}{\partial \varphi} + v_z \frac{\partial f(x,v)}{\partial z} + \frac{v_\varphi^2}{r}\frac{\partial f(x,v)}{\partial v_r} - \frac{v_r v_\varphi}{r}\frac{\partial f(x,v)}{\partial v_\varphi}$$
$$= \int \sigma g [f(x, v_1')f(x, v') - f(x, v_1)f(x, v)]dv_1 d\Omega. \tag{2.1}$$

Here $\sigma d\Omega$ is the differential collision cross-section; $g = |v - v_1|$ is the relative velocity; v, v_1, v', and v_1' are the velocities of two colliding molecules before and after collision. The boundary conditions are as follows: it is assumed that particles that leave the source surface are in thermal equlibrium with the source, i.e., the particle velocity is governed by the Maxwellian distribution

$$f(r = r_0, v) = \frac{\Phi}{2\pi}\left(\frac{m}{kT_w}\right)^2 \exp\left\{-\frac{mv^2}{2kT_w}\right\}, \quad v_r > 0, \tag{2.2}$$

where m is the mass of the particle; T_w is the temperature of the source wall; Φ is the flux of evaporating particles; k is the Boltzmann constant. Since evaporation proceeds in vacuum, the second boundary condition takes the form

$$f(r \to \infty, v) \equiv 0. \tag{2.3}$$

The hard sphere model of [32] is applied to describe the interaction between molecules. Problem (2.1)–(2.3) is solved using dimensionless variables. To normalize velocity, temperature, and mass, one can use, respectively, $\sqrt{2kT_w/m}$, T_w, and m. The gas density and mean free path λ_0 that correspond to the state of the gas at temperature T_w and flux Φ, are taken as the unit density and the unit length, respectively. The variable $\lambda_0\beta$ [$\beta = \sqrt{m/(2kT_w)}$] is used to nondimensionalize time. In this case, the nondimensional formulation of the problem has only one parameter, namely, the Knudsen number

$$\mathrm{Kn} = \lambda_0/2r_0. \tag{2.4}$$

The method of direct statistical modeling [32, 33] was used to solve the problem formulated. This method uses cylindrical cells whose radial size was decreased as the source was approached. For molecules that return to the surface, two cases are considered: 1) complete absorption; 2) diffuse reflection.

A cylindrical surface that absorbs all incoming particles is located at distance $d/2$ from the center of the source. This distance changes according to Kn. For every value of Kn, the distance d was chosen so as to prevent the influence of the absorbing surface on the flow behavior near the source. All the basic hydrodynamic moments of the distribution function were determined in the calculations. The calculations were carried out for a wide range of Kn values defined by (2.4): from 0.00005 to free-molecular flow. One of the objects of this work was to investigate the behavior of the Knudsen layer with changes of rarefaction conditions. In the foregoing, the boundary of the Knudsen layer was defined according to the conditions of applicability of continuous approaches. However, the method of direct statistical modeling does not allow this boundary to be found exactly because of the probabilistic character of this method, which leads to statistical fluctuations of the calculated moments. From this point of view, the Knudsen layer boundary can be determined with some accuracy δ, as in [34]. We specify δ in the form

$$\delta = \max\left[\frac{|T_r - T|}{T}, \frac{|T_\varphi - T|}{T}, \frac{|T_z - T|}{T}\right].$$

Here $T = (T_r + T_z + T_\varphi)/3$.

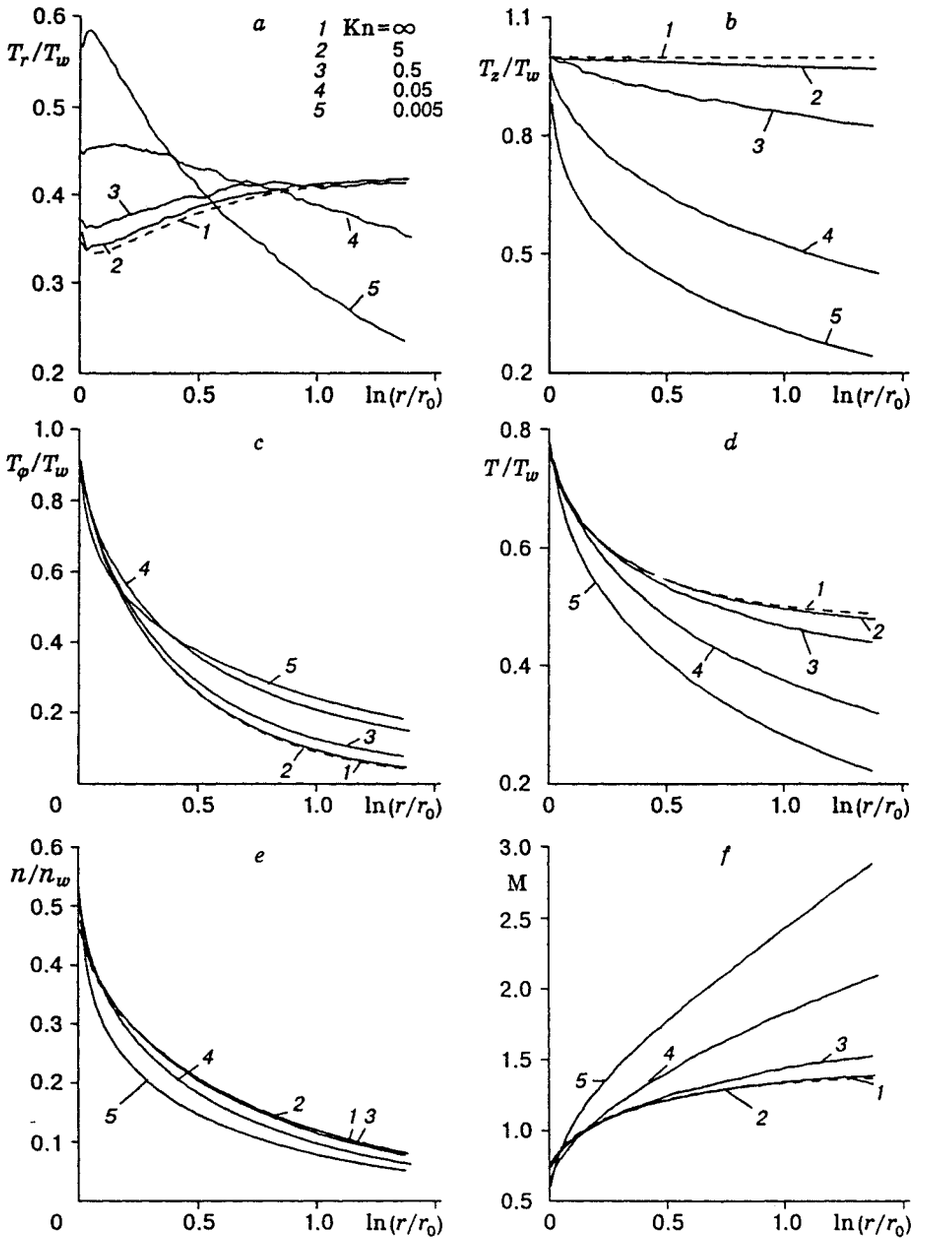
a
b
c
d
e
f
1 Kn=∞
2 5
3 0.5
4 0.05
5 0.005
T_r/T_w
T_z/T_w
T_φ/T_w
T/T_w
n/n_w
M
$\ln(r/r_0)$

Fig. 2

Thus, δ defines the deviation of the coordinate components of the temperature from the mean value. The dependence of δ and of the dispersions of the studied moments on the distribution function can be found by a simple procedure. Below, therefore, we use a value of δ that permits the clearest differentiation between statistical fluctuations and thermal nonequlibrium in the Knudsen layer.

3. Results of Investigation. The range from free-molecular to continuous flow regimes was studied. A rigorous formulation of the problem of stationary free-molecular flow from a cylindrical source into vacuum makes no physical sense. The fact is that the cylindrical expansion of the flow into infinity has no stationary solution. At a given constant flux, the initial stage of scattering of real molecules even into absolute vacuum at any but finite Knudsen number is accompanied by molecular collisions, which are significant for energy redistribution.

Calculated radial distributions of the parameters T_r, T_z, T_φ, T, n, and M for Kn = 0.005, 0.05, 0.5, and 5 are shown in Fig. 2. The analytical results of [26] for free-molecular flow are also shown here. Note the characteristic nonmonotony of the curve for T_r at distances $r/r_0 < 1.06$ as Kn $\to \infty$. This is due to the screening effect of the surface bending, as was indicated by Sone [35], who brought into use the concept of the S-layer. The limiting parameters for free-molecular flow near the cylinder surface are of interest: $T_w = 0.787$, $T_{\varphi w} = 1.000$, $T_{rw} = 0.363$, $u_{rw} = 0.564$, and also $T_{r\,\min} = 0.334$ and $u_\infty = 0.886$. The value $\mathrm{M} = u/\sqrt{\gamma RT} = 1$ has the coordinate $(r/(d/2))_{\mathrm{M}=1} = 1.16$. This abstract value, expressed as a Mach number, shows the limiting location of the sonic surface in the expanding flow. The asymptotic values of the parameters on the sonic surface for Kn $\to 0$ are given in [36].

Let us examine the behavior of the distribution of the parameters near the cylinder as Kn increases. Note that the changes in the distribution of the parameters are most significant as Kn changes in the range from 0.5 to 0.005. For T_r (see Fig. 2a), this change is qualitative. At large Kn values, the function $T_r(r)$ has a maximum, while at small values it has a minimum. The maximum remains even at smaller Kn values (Kn < 0.005) while its location comes closer to the cylinder. While the minimum, as one can see from Fig. 2a, is associated with a purely geometric effect, the maximum results from a relaxation process within the Knudsen layer. The collision "game" of molecules is responsible for the peculiarity in the behavior of T_φ (see Fig. 2c), which involves a nonmonotonic change in the azimuthal component of temperature with a change in Kn in the flow region under study. The change in $T_\varphi = F(r)$ with Kn is less pronounced than for T_r and T_z. The changes in the density and Mach number are shown in Figs. 2e and 2f. One can clearly see a correlation between these two parameters with a decrease in Kn: the density rises and the velocity falls off near the cylinder, while the density decreases as the velocity rises far away from the surface.

The calculation allows one to determine the number of particles returning to the evaporation surface from the region around the cylinder. These particles can either be condensed at the surface or reflected. The structure of the Knudsen layer depends on the flow intensity of returning particles and vice versa. Almost identical functions $j = i_r/i_e = F(\mathrm{Kn})$ are shown in Fig. 3 for two cases: condensation with $\alpha = 1$ and diffuse reflection. Here i_r is the total flux of returning particles and i_e is the total flux of evaporated (injected) and reflected particles. One can see that as Kn $\to 5 \cdot 10^{-5}$ the portion of returning particles approaches asymptotically a constant value. Assuming that the evaporation coefficient is equal to 1, the conclusion can be drawn that Fig. 3 describes quantitatively the mass flux decrease with decreasing Kn in comparison with the mass flux according to the Hertz–Knudsen law. For reference, the data of [19, 26] are also given in Fig. 3. Note that the rate of change in the portion of particles returning to the surface has a maximum at Kn ~ 0.05.

The radial coordinate of the sonic surface (in free paths) counted from the cylinder surface is shown in Fig. 4. Note that, for $\alpha = 1$, the sonic surface at Kn = 0.005 is located at a distance of ≈ 3.5 mean free paths, i.e., in the usual boundary region of the Knudsen layer. At Kn = 0.00005, it is located at approximately 30 mean free paths. For Kn = 0.002, the thickness of the Knudsen layer was found in [26] to be equal to roughly 15 mean free paths that were determined from the source parameters. This leads to the conclusion that the sonic surface location cannot be unambigously related to the number of free paths that characterizes the Knudsen layer thickness. The character of the dependences of the sonic surface location obtained for $\alpha = 0$ and 1 in Fig. 4 is not indicative of the near asymptote (stabilization of the sonic surface location) with decreasing Kn. At small values of Kn, the sonic surface is shifted into the continuous flow zone, more exactly,

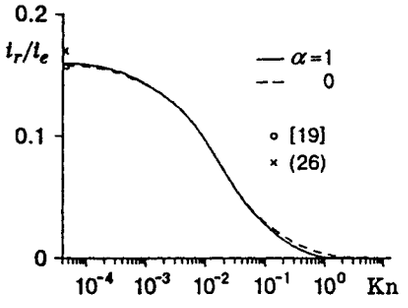

Fig. 3

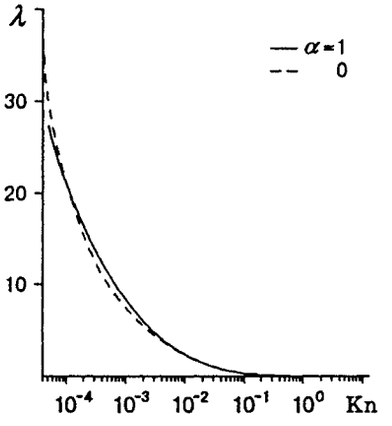

Fig. 4

into the zone remote from the near-wall relaxation.

Indeed, the boundary of significant influence of nonequlibrium near the cylinder surface that corresponds to $\mathrm{Kn}_l = 0.01$ is located, for $\mathrm{Kn} = 0.00005$, at a distance of 4.4 free paths from the source surface, i.e., much closer than the sonic surface. This can be due only to the presence of heat flux and friction in front of the sonic surface. This follows from the law of transition from subsonic to supersonic flow that was formulated by L. A. Vulis in 1950 and is known as the law of reversion of action [37]:

$$(\mathrm{M}^2 - 1)\frac{du}{u} = \frac{dr}{r} - \psi dq - \theta dL - \frac{d(rnu)}{rnu} - \xi dA. \tag{3.1}$$

Here ξ, ψ, and θ are the coefficients, which are known from thermodynamics, of, respectively, the thermal dq, dissipative dL, and mechanical (in the sense of work done) dA actions. Within the zone between the Knudsen layer and the sonic surface for the one-dimensional flow under study we assume that $dnu = 0$ and $dA = 0$. In this case, only terms expressing the geometric, thermal and dissipative actions remain on the right side of (3.1).

The effects of thermoconductivity and viscosity can be evaluated from the change in the stagnation parameters in the zone between point 1, where $\mathrm{Kn}_l = 0.01$, and point 2, where $\mathrm{M} = 1$. For $\mathrm{Kn} = 0.00005$, from the calculations we have all the necessary data to determine the stagnation parameters T_0 and P_0 at points 1 and 2 using the gasdynamics relations

$$T_0 = T(1 + \frac{\gamma - 1}{2}\mathrm{M}^2), \quad P_0 = P(1 + \frac{\gamma - 1}{2}\mathrm{M}^2)^{\gamma/(\gamma-1)}$$

assuming that the gas came to points 1 and 2 from stagnation conditions by isentropic expansion. For the selected gas space within dihedral angle $\Delta\varphi$, the friction force due to viscosity effects is determined from the balance between the sums of forces governing the change in the momentum flux:

$$\rho_1 u_1^2 r_1 \Delta\varphi - \rho_2 u_2^2 r_2 \Delta\varphi + p_1 r_1 \Delta\varphi - p_2 r_2 \Delta\varphi + \Delta\varphi \int_{r_1}^{r_2} p\, dr - X_{\mathrm{fr}} = 0.$$

The effect of this force can be estimated by relating the friction force to a characteristic surface, for example, $r_1\Delta\varphi$ at point 1:

$$\frac{X_{\mathrm{fr}}}{r_1\Delta\varphi} = \rho_1 u_1^2 - \rho_2 u_2^2 \frac{r_2}{r_1} + p_1 - p_2 \Delta\varphi \frac{r_2}{r_1} + \frac{1}{r_1}\int_{r_1}^{r_2} p\, dr. \tag{3.2}$$

The stagnation temperature increase $\Delta_{1-2}T_0$ is a small value of about 10^{-3}, which does not allow one to study the character of the T_0 variation between points 1 and 2; moreover, the difference between the temperatures

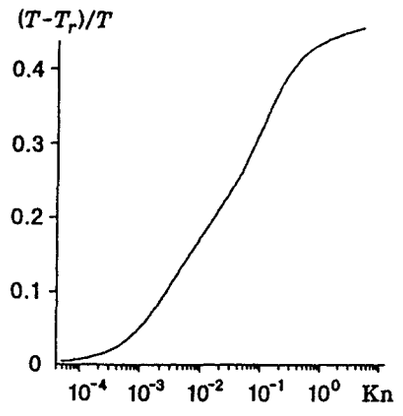

Fig. 5

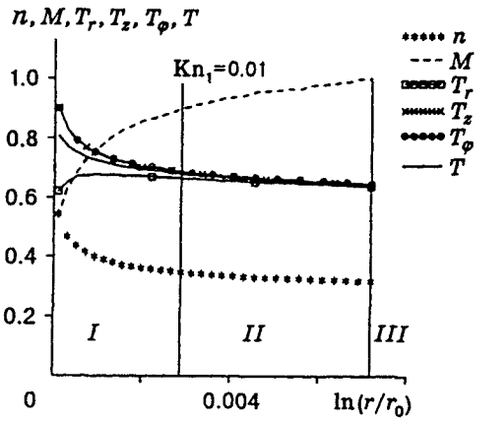

Fig. 6

T_r and T in this region is of the same order. The stagnation pressure between points 1 and 2 changes definitely and noticeably: it falls off by a factor of 0.03. From (3.2) one can obtain $X_{\text{fr}}/(r_1\Delta\varphi) = 0.02$, which corresponds to the level of full pressure losses.

The analysis performed for nonisentropic flow upstream the sonic surface is of course correct only within the framework of continuous medium theory for $\text{Kn}_l < 0.01$.

Since a sonic surface can generally exist either rarefied or within continuous flow, it would be interesting to describe the nonequlibrium state of this surface by the function $(T - T_r)/T = F(\text{Kn})$ (see Fig. 5). This dependence has limiting asymptotes. Specifying the value $\delta = 1 - T_r/T = 10^{-2}$, one can contend that at $\text{Kn} = 0.0005$ the sonic surface is located in the continuous flow zone. The shape of the curve in Fig. 5 suggests that approximate coincidence of the boundary of the nonequlibrium zone with the sonic surface takes place at $\text{Kn} = 0.005$.

The spatial separation of the sonic surface and the Knudsen layer does not clarify the characteristics of the latter. In flow calculations using the Monte Carlo method no problem of the Knudsen layer exists and its thickness can be determined from the results of statistical modeling with an approximation such that statistical fluctuations can be differentiated from nonequilibrum.

For a more complete description of the formation process of the sonic surface, Fig. 6 shows the density, velocity, and temperature curves in the zone between the cylinder surface and the sonic surface for $\text{Kn} = 0.00005$, i.e., for bulk continuous flow. If continuous flow forms near the cylinder as a result of injection or evaporation into vacuum, the parameters under conditions of radial vapor/gas flow change successively: 1) at near-surface discontinuities, 2) in the kinetic zone at $\text{Kn}_l > 0.01$, in the subsonic flow zone at $\text{Kn}_l < 0.01$, and in the supersonic zone (zones I–III in Fig. 6).

Such a character of flow from convex surfaces into vacuum is common for bodies of any shape. If the injected gas (vapor) expands into the background with some pressure that prevents transition through the sonic speed, all the above considerations are valid for the absence of the supersonic flow zone.

4. Conclusions. In this paper, an attempt is made to perform a systematic analysis of the nonequilibrium zone of gas expansion from a cylindical source with evaporation or injection into vacuum under conditions ranging from free-molecular to continuous flow using direct statistical modeling by the Monte Carlo method.

Variations in the basic parameters during expansion are presented and the mechanism of formation of the sonic surface is established. It is shown that at small Knudsen numbers the sonic surface is located within the continuous flow zone.

The results obtained make it possible to solve the problem of evaporation or injection from finite surfaces in new physical formulations taking into account the internal degrees of freedom of evaporating molecules and condensation in supersaturated flow, and also to analyze the flow of inhomogeneous gases.

The authors would like to thank P. A. Skovorodko for helpful discussions of the results of the investigation.

This work was supported by the Russian Foundation for Fundamental Research (Grant 95-01371a).

REFERENCES

1. R. Ya. Kucherov and L. É. Rikenglaz, "On the hydrodynamic boundary conditions in evaporation and condensation," *Zh. Eksp. Teor. Fiz.*, **37**, No. 1, 125–126 (1959).
2. A. J. Patton and G. S. Springer, "A kinetic theory description of liquid vapor phase change," in: Int. Symp. on Rarefied Gas Dynamics. Cambridge, 1968. Acad. Press, New York, **2** (1969), pp. 1497–1501.
3. P. N. Shankar and F. E. Marble, "Kinetic theory of transient condensation and evaporation at a plane surface," *Phys. Fluids*, **14**, No. 3, 510–516 (1971).
4. R. Mager, G. Adomeit, and G. Wotberg, "Theoretical and experimental investigation of the strong evaporation of solids," in: Int. Symp. on Rarefied Gas Dynamics. Pasadena, 1988. AIAA, Washington (1989), pp. 460–471.
5. T. Ytrehus, "Theory and experiment on gas kinetics in evaporation," in: Int. Symp. on Rarefied Gas Dynamics. Aspen, July, 1976. AIAA, New York (1977), pp. 1197–1212.
6. F. G. Cheremisin, "Rarefied gas flow between emitting and absorbing infinite parallel surfaces," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 2, 176–178 (1972).
7. S. M. Yen, "Solutions of the kinetic equation for the nonequilibrium gas flow between emitting and absorbing surfaces," in: Int. Symp. on Rarefied Gas Dynamics, Gottingen, 1974. DFVLR-Press, Porz-Wahu (1974), Vol. 1, pp. A15/1–A15/10.
8. J. W. Cipolla, Jr., "Transition regime droplet growth and evaporation: an integrodifferential variational approach," in: Int. Symp. on Rarefied Gas Dynamics. Pasadena, 1988. AIAA, Washington (1989), pp. 434–442.
9. Y. P. Pao, "Application of kinetic theory to the problem of evaporation and condensation," *Phys. Fluids*, **14**, No. 2, 306–312 (1971).
10. Y. P. Pao, "Temperature and density jumps in the kinetic theory of gases and vapors," *Phys. Fluids*, **14**, No. 7, 1340–1346 (1971).
11. S. E. Siewert and J. R. Thomas, Jr., "Half-space problem in the kinetic theory of gases," *Phys. Fluids*, **16**, No. 9, 1557–1559 (1973).
12. T. Matsushita, "An analytical study of the problem of evaporation and condensation using the finite element method," in: Int. Symp. on Rarefied Gas Dynamics. Gottingen, 1974. DFVLR-Press, Porz-Wahu (1974), Vol. 2, pp. F3/1–F3/12.
13. N. M. Kogan and N. K. Makashev, "Role of the Knudsen layer in the theory of heterogeneous reactions and in flows with surface reactions," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 6, 3–6 (1971).
14. S. I. Anisimov and A. H. Rakhmatulina, "Vapor expansion dynamics in evaporation into vacuum," *Zh. Eksp. Teor. Fiz.*, **64**, No. 3, 869–876 (1973).
15. N. K. Makashev, "Strong recondensation in one- and two-component rarefied gas at arbitrary values of the Knudsen number," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 5, 130–138 (1972).
16. A. M. Bishaev and V. A. Rykov, "Recondensation of the monatomic layer at small values of the Knudsen number," *Zh. Vychisl. Mat. Mat. Fiz.*, **18**, No. 3, 709–717 (1978).
17. M. N. Kogan and N. K. Makashev, "Role of the Knudsen layer in the theory of heterogeneous reactions and in flows with surface reactions," *Fluid Dyn.*, **6**, 913–920 (1974).
18. Y. Onishi, "Nonlinear analysis for evaporation and condensation of a vapor–gas mixture between the two plane condensed phases. Part 1: Concentration of inert gas 0(1)," Int. Symp. on Rarefied Gas Dynamics, Pasadena, 1988. AIAA, Washington (1989), pp. 470–491.
19. C. J. Knight, "Evaporation from cylindrical surface into a vacuum," *J. Fluid Mech.*, **75**, Part 3, 469–486 (1976).

20. Y. Sone, K. Aoki, H. Sugimoto, and T. Yamada, "Steady evaporation and condensation on a plane condensed phase," *Theor. Appl. Mech.* (Bulgaria), **3**, 89–97 (1988).
21. S. I. Anisimov, "On the evaporation of a metal absorbing laser radiation," *Zh. Eksp. Teor. Fiz.*, **54**, No. 1, 339–342 (1968).
22. A. V. Luikov, T. L. Perelman, and S. I. Anisimov, "Evaporation of a solid into a vacuum," *Int. J. Heat Mass Transfer*, **14**, 177–184 (1971).
23. J. W. Jr. Cipolla, H. Lang, and S. K. Loyalka, "Kinetic theory of condensation and evaporation," *J. Chem. Phys.*, **61**, No. 1, 69–77 (1974).
24. D. A. Labuntsov and A. P. Kryukov, "Analysis of intensive evaporation and condensation," *Int. J. Heat Mass Transfer.*, **22**, No. 7, 989–1002 (1979).
25. H. Sugimoto and Y. Sone, "Numerical analysis of steady flows of a gas evaporating from its cylindrical condensed phase on the basis of kinetic theory," *Phys. Fluids, A*, **4**, No. 2, 419–440 (1992).
26. D. Sibold and H. M. Urbassek, "Kinetic study of evaporation flows from cylindrical jets," *Phys. Fluids, A*, **3**, No. 5, 870–878 (1991).
27. M. Faubel, S. Schlemmer, and J. P. Toennies, "A molecular beam study of the evaporation of water from a liquid jet," *Z. Phys., D*, **10**, 269 (1988).
28. M. Murakami and K. Oshima, "Kinetic approach to the transient evaporation and condensation problem," in: Int. Symp. on Rarefied Gas Dynamics. Gottingen, 1974. DFVLR-Press, Porz-Wahu (1974), pp. F6/1–F6/10.
29. A. A. Abramov, "Solution of the problem of strong evaporation of a monatomic gas by the Monte Carlo method," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 1, 185–188 (1984).
30. St. Radev and St. Stefanov, "Monte Carlo simulation of vapor–gas cylindrical coutter flow," *Theor. Appl. Mech.* (Bulgaria), No. 3, 70–77 (1988).
31. M. N. Kogan, *Rarefied Gas Dynamics* [in Russian], Nauka, Moscow (1967).
32. G. Bird, *Molecular Gasdynamics* [Russian translation], Mir, Moscow (1981).
33. M. S. Ivanov and S. V. Rogazinskii, "Efficient schemes of direct statistical modeling of rarefied gas flows," *Mat. Model.*, **1**, No. 7, 130–145 (1989).
34. D. Sibold and H. M. Urbassek, "Monte Carlo study of the Knudsen layers in evaporation from elemental and binary media," *Phys. Fluids, A*, **5**, No. 1, 243–256 (1993).
35. Y. Sone, "New kind of boundary layer over a convex solid boundary in a rarefied gas," *Phys. Fluids*, **16**, 1422–1424 (1973).
36. K. Aoki and Y. Sone, "Gas flows around the condensed phase with strong evaporation or condensation. Fluid dynamic equation and its boundary condition on interface and their application," in: *Advances in Kinetic Theory and Continuum Mechanics*: Proc. of a Symp. Held in Honor of Professor H. Cabannes, Paris, July 6, 1990. Springer–Verlag, Berlin (1991), pp. 43–54.
37. G. N. Abramovich, *Applied Gas Dynamics* [in Russian], Nauka, Moscow (1969).